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Renormalization group equation for interacting Thirring fields in dimensional regularization scheme

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Abstract. The dynamics of two interacting Thirring fields has been investigated within the dimensional regularization framework. The coupling constants are renormalized in the same way as observed in the non-perturbative approach of Ansel'm *et al.* Functions $\beta_i(g_1, g_2, g_3)$ and $\gamma_i(g_1, g_2, g_3)$, pertaining to the stability and anomalous behaviour of the problem, are computed up to third order in the coupling parameters. With the help of these, subsidiary non-linear differential equations of the renormalization group are studied in $2-\epsilon$ dimension. The results show up some peculiar features of the theory: a zero of $\beta_i(g_1, g_2, g_3)$ corresponding to $g_2 \approx \alpha\sqrt{\epsilon}$, a characteristic of ϕ^3 theory. The scale invariant limit is reached when $g_2 \to 0$ (i.e. the two Thirring fields are decoupled) and also when $g_1 = xg_2 = g_3$, where x is a root of $2x^3+2x^2-1=0$. The branch-point zero makes the transition to the $\epsilon \to 0$ limit non-unique. The anomalous dimensions are obtained and seen to match that of the Dashen–Frishman model. The existence of a non-trivial scale invariant limit distinguishes the model from many simple field theories.

1. Introduction

The study of asymptotic behaviour in quantum field theory in the light of present day experimental findings has been organized only recently, since the advent of the Callan (1970) and Symanzik (1971) equations and renormalization group (Bogoliubov and Shirkov 1900) equations. Though the asymptotic behaviour can only be assessed when the theory is exactly solvable, the Callan–Symanzik (CS) and renormalization group (RG) equations provide much information, not obtainable in each order of perturbation theory (as almost all the theories are not exactly solvable). One of the most interesting situations is presented by the Thirring model (exactly solvable) (Hagen 1967) and its SU(N) generalization, the Dashen–Frishman (DF) model (Dashen and Frishman 1973). The solvability of the Thirring model has been critically examined by Hagen (1967) in the light of both the CS and RG equations while the β and γ functions of the DF model have been computed and the corresponding stability problem has been discussed by Dashen and Frishman (1973). Intermediate between the two is the study of Mayer and Geieke (1973) who have shown that the usual Thirring model considered in $2-\epsilon$ dimension does not possess an expansion in ϵ. But a further problem which is well worth studying is that of two Thirring fields (i.e. spin-$\frac{1}{2}$ particles in two dimensions) interacting with each other as prescribed by the following Lagrangian:

$$L = \bar{\psi}(\gamma \cdot \partial + m)\psi + \bar{\chi}(\gamma \cdot \partial + m)\chi + g_1(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) + g_2(\bar{\psi}\gamma_\mu\psi)(\bar{\chi}\gamma_\mu\chi) + g_3(\bar{\chi}\gamma_\mu\chi)(\bar{\chi}\gamma_\mu\chi) \quad (1)$$

Incidentally we may remark that this model was considered by Ansel'm (1959) and Dyatlov *et al* (1957) with an eye to asymptotic behaviour in an exact integral equation formalism. But for this situation, regarding stability, scale invariance and anomalous dimensions of the fields, we have made an elaborate study in the dimensional regularization scheme of 't Hooft and Veltman (1972). The reason behind using dimensional regularization rather than the usual subtractive procedure of Bogoliubov and Parasiuk (1957) and Zimmermann (1970) is that one of our basic intentions is to explore the possibility of an ϵ expansion of the theory.

2. Formulation

Lagrangian (1) immediately suggests that the net effect of figures 1 and 2 is to renormalize the mass of both the fields ψ and χ, while figures 3, 4 and 5 contribute to renormalizing the coupling constants g_1, g_2, g_3. Figure 1(a) contributes

$$\gamma_\alpha\gamma_\alpha \int \mathrm{i}g_1 \frac{(\mu^2)^{1-(n/2)}}{(2\pi)^n} \frac{\mathrm{d}^n k}{\not{K}-m} = -\frac{g_1 m}{\pi}\frac{1}{2-n}. \tag{2}$$

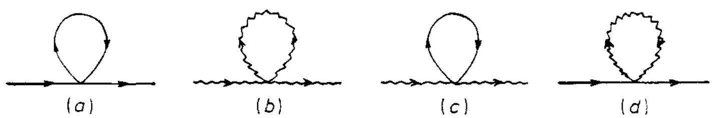

Figure 1. One loop (first order in g) mass renormalization. Full lines represent ψ fields and wavy lines represent χ fields in figures 1–5.

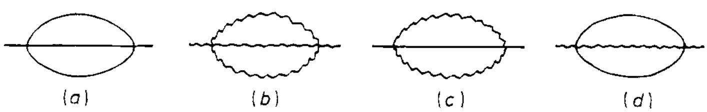

Figure 2. Two loop (second order in g) mass renormalization.

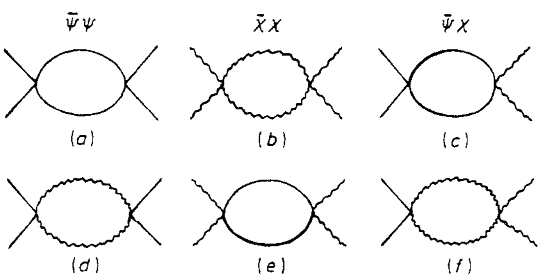

Figure 3. One loop coupling constant renormalization.

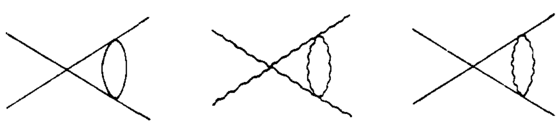

Figure 4. Some diagrams of two loop (third order in g) renormalization.

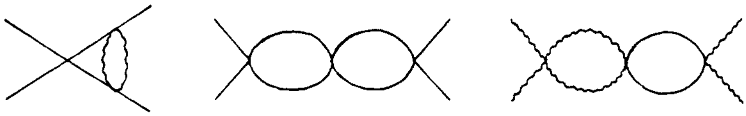

Figure 5.

Similarly for figures 1(b), (c) and (d). In second order the two loop diagrams (figure 2) contribute

$$\int \frac{\mathrm{Tr}[\gamma^\nu(\not k+m)\gamma_\mu(m-\not q)][\gamma^\nu(\not p-\not k+\not q+m)\gamma_\mu]\,\mathrm{d}k\,\mathrm{d}q}{(k^2-m^2)(q^2-m^2)[(p-k+q)^2-m^2]} + \int \frac{\gamma^\nu(\not k+m)\gamma_\mu(-\not q+m)\gamma^\nu(\not p+m-\not k+\not q)\gamma_\mu\,\mathrm{d}q\,\mathrm{d}k}{(k^2-m^2)(q^2-m^2)[(p-k+q)^2-m^2]}. \tag{3}$$

One of the prototypes of the integrals occurring in (3) is

$$\int \frac{\not q\not k\,\mathrm{d}q\,\mathrm{d}k}{(k^2-m^2)(q^2-m^2)[(p-k+q)^2-m^2]} = 2!\int \frac{\not q\not k\,\mathrm{d}q\mathrm{d}kx\,\mathrm{d}x\,\mathrm{d}z}{\alpha^3\{q^2+2q(p-k)(xz/\alpha)+[k^2x+(p^2-2p\,.\,k)x^2]/\alpha\}^3}.$$

The integrations over q, k can be performed and one can write the integral in the following form:

$$-\Gamma(2-n)\int_0^1 x^{(n/2)-1}(1-Px)^{2-n}(1-Qx)^{-[3-(n/2)]}z\,\mathrm{d}z\,\mathrm{d}x \qquad P=1-z, \qquad Q=1-z+z^2 \tag{4}$$

which can be explicitly evaluated by noticing that

$$\int_0^1 U^{\alpha-1}(1-U)^{\gamma-\alpha-1}(1-Ux)^{-\beta}(1-Uy)^{-\beta'} \equiv \frac{\Gamma(\alpha)\Gamma(\gamma-\alpha)}{\Gamma(\gamma)}F_1(\alpha,\beta,\beta',\gamma,x,y) \tag{5}$$

where $F_1(\alpha, \beta, \beta', \gamma, x, y)$ is the generalized hypergeometric function of six arguments. So equation (4) reduces to

$$\Gamma(2-n)\int z\,\mathrm{d}z\frac{\Gamma(\frac{1}{2}n)\Gamma(1)}{\Gamma(\frac{1}{2}n+1)}F_1(\tfrac{1}{2}n, n-2, 3-\tfrac{1}{2}n, 1+\tfrac{1}{2}n, P, Q)$$

which in the limit $n\to 2$ yields simply $-\frac{1}{2}\Gamma(2-n)$. Taking care with the functions of (2π) occurring at each vertex we find that the contribution of figure 2(a) is $-g^2/4\pi^2(2-n)$.

Turning now to the coupling constant renormalization, we first consider figures 3(a), 3(b) etc. The most important point to notice about figure 3(a) is that its divergent contribution is exactly annulled by the contribution of its crossed graph (figure 3(b)). It is interesting to note that it is this mechanism which prevented the coupling constant in the usual Thirring model from being renormalized. So in second order we have no divergent contribution to the coupling constant renormalization. Thus we pass over to the third-order diagrams depicted in figures 4 and 5. The contribution of the diagrams of figure 4 is given by

$$\int \frac{\gamma_\mu \mathrm{Tr}[\gamma^\mu(\not{K}_1+m)\gamma^\sigma(\not{p}_3+\not{K}_1-\not{K}+m)](\gamma^\sigma(\not{K}+m)\gamma^\lambda(\not{p}-\not{K}+m)\gamma^\lambda)}{(k^2-m^2)(k_1^2-m^2)[(p-k)^2-m^2][(p_3+k_1-k)^2-m^2]}$$

$$+\int \frac{\gamma_\mu(\gamma^\mu(\not{K}_1+m)\gamma^\sigma)\,\mathrm{Tr}[(\not{p}_3+\not{K}_1-\not{K}+m)\gamma^\sigma(\not{K}+m)\gamma^\lambda(\not{p}-\not{K}+m)\gamma^\lambda]}{(k^2-m^2)(k_1^2-m^2)[(p-k)^2-m^2][(p_3+k-k_1)^2-m^2]}$$

$$+\int \frac{\gamma_\mu[\gamma^\mu(\not{K}_1+m)\gamma^\sigma(\not{p}_3+\not{K}_1-\not{K}+m)\gamma^\sigma(\not{K}+m)\gamma^\lambda(\not{p}-\not{K}+m)\gamma^\lambda]}{(k^2-m^2)(k_1^2-m^2)[(p-k_1^2-m^2][(p_3+k-k_1)^2-m^2]}. \tag{6}$$

Again the integrals involved can be written in the form (5) and we find the following contribution from figure 4(a):

$$g_1^3\left(\frac{1}{4\pi^4(2-n)^2}-\frac{\mathrm{i}\ln(p^2/\mu^2)}{4\pi^4(2-n)}+\frac{3\mathrm{i}\gamma}{4\pi^4(2-n)}\right) \tag{7}$$

which is to be combined with the counter term diagram, whose contribution is equal to

$$-\frac{2\mathrm{i}g_1^3}{4\pi^4(2-n)^2}+\frac{\mathrm{i}g_1^3\ln(p^2/\mu^2)}{4\pi^4(2-n)}-\frac{\mathrm{i}g_1^3\gamma}{4\pi^4(2-n)} \tag{8}$$

so figure 4(a) plus counter term yields

$$-\frac{\mathrm{i}g_1^3}{4\pi^4}\left(\frac{1}{(2-n)^2}-\frac{2\gamma}{2-n}\right). \tag{9}$$

The contributions of other graphs in the set are similar except for the coupling factors. Again, the contribution of a chain diagram in third order (figure 5) is

$$-\frac{\mathrm{i}g_1^3}{(2\pi)^{3n}}n^3\left(\int\frac{\mathrm{d}q}{(\not{q}-m)(\not{p}-\not{q}-m)}\right)^2=\frac{\mathrm{i}g_1^3}{2\pi^4}\frac{1}{(2-n)^2}+\frac{\mathrm{i}g_1^3\gamma}{2\pi^4(2-n)^2}-\frac{\mathrm{i}g_1^3\ln(p^2/\mu^2)}{2\pi^4(2-n)} \tag{10}$$

which, when combined with its counter term contribution

$$-\frac{\mathrm{i}g_1^3}{\pi^4(2-n)^2}-\frac{\mathrm{i}g_1^3\gamma}{2\pi^4(2-n)}+\frac{\mathrm{i}g_1^3}{2\pi^4}\frac{\ln(p^2/\mu^2)}{2-n}$$

yields

$$-\frac{\mathrm{i}g_1^3}{2\pi^4(2-n)^2}. \tag{11}$$

So collecting these results we have the following connection between renormalized and bare coupling constants:

$$g_1^{\mathrm{B}}=\mu^{2-n}\bigg(g_1^{\mathrm{R}}+\mathrm{i}\frac{g_1g_2^2\gamma}{8\pi^4(2-n)}+\frac{\mathrm{i}g_2^3\gamma}{8\pi^4(2-n)}-\frac{\mathrm{i}g_1g_2^2}{4\pi^4(2-n)^2}$$
$$-\frac{\mathrm{i}g_2^3}{4\pi^4(2-n)^2}-\frac{\mathrm{i}g_1g_2^2}{2\pi^4(2-n)^2}-\frac{\mathrm{i}g_3g_2^2}{2\pi^4(2-n)^2}+\cdots\bigg)$$

$$g_2^B = \mu^{2-n}\left(g_2^R + \frac{\Sigma}{8\pi^4(2-n)} - \frac{\Sigma}{4\pi^4(2-n)^2} - \frac{\Lambda}{2\pi^4(2-n)^2} + \dots\right) \tag{12}$$

$$\Sigma = \mathrm{i}(g_3 g_2^2 + g_2 g_1^2 + g_2 g_3^2 + g_2^3 + g_1 g_2^2)\gamma + \dots$$

$$\Lambda = \mathrm{i}(g_2 g_1^2 + g_2 g_3^2 + g_1 g_2 g_3 + g_2^3) + \dots \tag{13}$$

$$g_3^B = \mu^{2-n}\left(g_3^R + \mathrm{i}\frac{g_3 g_2^2 \gamma}{8\pi^4(2-n)} + \frac{\mathrm{i} g_2^3 \gamma}{8\pi^4(2-n)} - \frac{\mathrm{i} g_3 g_2^2}{4\pi^4(2-n)^2} - \frac{\mathrm{i} g_2^2}{4\pi^4(2-n)^2} - \frac{\mathrm{i}(g_3 g_2^2 + g_1 g_2^2)}{2\pi^4(2-n)^2}\right). \tag{14}$$

It is interesting to note that as the mixing interaction $g_2 \to 0$ we get $g_1^B = g_1^R$ and $g_3^B = g_3^R$ for the decoupled Thirring fields. In this connection it is worth mentioning that similar results were obtained by Ansel'm for the renormalized coupling constants from Schwinger–Dyson type integral equations in the asymptotic region.

3. Group equations

Following the usual technique of varying the unit of mass introduced in 't Hooft's method of renormalization, we obtain

$$\left(\mu\frac{\partial}{\partial\mu} + \sum_k \beta^k(g, m, \mu)\frac{\partial}{\partial g_k} - \sum \gamma^i(g, m, \mu)\right)\Gamma^N = 0 \tag{15}$$

where g without a suffix stands for the full set (g_1, g_2, g_3) and β^k, γ^i are determined according to

$$\beta^k = \mu\frac{\partial g_k}{\partial\mu}\bigg|_{\text{at bare values}}, \qquad \gamma^i = \frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_3^i\bigg|_{\text{at bare values}} \tag{16}$$

Analyticity in the complex dimensional plane implies that the renormalization constants γ^i, β^k are all expressible as Laurent series, and computing in the manner of 't Hooft and Veltman (1972), we get

$$\beta^1(g, m, \mu) = \epsilon g_1 - \frac{\mathrm{i}\gamma}{4\pi^4} g_2^2(g_1 + g_2)$$

$$\beta^2(g, m, \mu) = \epsilon g_2 - \frac{\mathrm{i}\gamma}{4\pi^4} g_2[g_2^2 + g_1(g_1 + g_2) + g_3(g_3 + g_2)] \tag{17}$$

$$\beta^3(g, m, \mu) = \epsilon g_3 - \frac{\mathrm{i}\gamma}{4\pi^4} g_2^2(g_2 + g_3)$$

$$\gamma_1 = -\frac{m}{\pi}(g_1 + g_2) - \frac{1}{2\pi^2}(g_1^2 + g_2^2)$$

$$\gamma_2 = -\frac{m}{\pi}(g_3 + g_2) - \frac{1}{2\pi^2}(g_3^2 + g_2^2). \tag{18}$$

4. Solution and inferences

Solution of the group equation (15) by invoking the homogeneity condition

$$\left(\lambda\frac{\partial}{\partial\lambda}+\sum g_{\mathrm{R}}^{k}\frac{\partial}{\partial g_{\mathrm{R}}^{k}}+\mu\frac{\partial}{\partial\mu}-D\right)\Gamma=0$$

yields

$$\Gamma^{\mathrm{N}}(\lambda p_i, g^k; m, \mu) = \Gamma^{\mathrm{N}}(p, \bar{g}^k(\lambda), m/\lambda, \mu)\lambda^{\alpha}\exp\left(-\sum_{i}\int_{1}^{\lambda}\frac{\mathrm{d}x}{x}\gamma^{i}(\bar{g}^k(x), m/x, \mu)\right). \quad (19)$$

Here $\bar{g}^k(\lambda)$ are effective coupling constants defined by the equations

$$\lambda(\mathrm{d}\bar{g}^k(\lambda)/\mathrm{d}\lambda)=\beta^k(\bar{g}^k(\lambda), m/\lambda, \mu) \quad (20)$$

where β^k are the functions computed in equation (17). Equation (19) helps us obtain the asymptotic behaviour in a more rigorous manner than the usual order-by-order perturbation theory by making some assumptions about the functions β^k, $k=1,2,3$. The functions in turn govern the characteristic properties of the effective charges. The effective charges can be obtained by analysing the solutions of the non-linear differential equations (2). Written in full, these equations read

$$\lambda\frac{\mathrm{d}\bar{g}_1}{\mathrm{d}\lambda}=\epsilon\bar{g}_1-\frac{\mathrm{i}\gamma}{4\pi^4}\bar{g}_2^2(\bar{g}_1+\bar{g}_2) \quad (21a)$$

$$\lambda\frac{\mathrm{d}\bar{g}_2}{\mathrm{d}\lambda}=\epsilon\bar{g}_2-\frac{\mathrm{i}\gamma}{4\pi^4}[\bar{g}_2^2+\bar{g}_1(\bar{g}_1+\bar{g}_2)+\bar{g}_3(\bar{g}_3+\bar{g}_2)]\,\bar{g}_2 \quad (21b)$$

$$\lambda\frac{\mathrm{d}\bar{g}_3}{\mathrm{d}\lambda}=\epsilon\bar{g}_3-\frac{\mathrm{i}\gamma}{4\pi^4}\bar{g}_2^2(\bar{g}_2+\bar{g}_3). \quad (21c)$$

Subtracting (21*a*) and (21*c*), we get

$$\lambda\frac{\mathrm{d}}{\mathrm{d}\lambda}(\bar{g}_1-\bar{g}_3)=(\bar{g}_1-\bar{g}_3)\left(\epsilon-\frac{\mathrm{i}\gamma}{4\pi^4}\bar{g}_2^2\right). \quad (22)$$

This equation is identically satisfied by the solution $\bar{g}_1-\bar{g}_3=0$. In searching for simultaneous zeros of the three β functions, we have to solve

$$\begin{aligned}&\bar{g}_2[\bar{g}_2^2+\bar{g}_1(\bar{g}_1+\bar{g}_2)+\bar{g}_3(\bar{g}_2+\bar{g}_3)]-\epsilon\bar{g}_2=0\\&\bar{g}_1=\bar{g}_3\end{aligned} \quad (23)$$

which yields $\bar{g}_2=0$ or

$$2\bar{g}_1^2+2\bar{g}_2^2+2\bar{g}_1\bar{g}_2=\epsilon(4\pi^4/\mathrm{i}\gamma). \quad (24)$$

Also, $\beta_1=0$ gives

$$\bar{g}_1\bar{g}_2^2+\bar{g}_2^3-\epsilon\frac{4\pi^4}{\mathrm{i}\gamma}\bar{g}_1=0, \quad (25)$$

so we get $\bar{g}_1=\bar{g}_3=x\bar{g}_2$, where x is a solution of the equation $2x^3+2x^2-1=0$ and $\bar{g}_2$ is given by

$$\bar{g}_2^2[1+(1/x)]=\epsilon. \quad (26)$$

Thus, apart from a free decoupled theory corresponding to $g_2 = 0$, we can also have a non-trivial simultaneous zero of β^k given by equations (25) and (26). But the mere existence of zeros of β functions is not enough to clarify the physical characteristics of the problem. For a critical analysis, the corresponding stability problem has to be solved. If we denote $\partial\beta^k/\partial g_\sigma$, $\sigma = 1, 2, 3$, by β^k_σ then the problem is really the determination of the eigenvalues of the matrix β^k_σ,

$$\beta^k_\sigma = \begin{pmatrix} \epsilon - \frac{i\gamma}{4\pi^4}\bar{g}_2^2 & -\frac{i\gamma}{4\pi^4}(2\bar{g}_1\bar{g}_2 + 3\bar{g}_2^2) & 0 \\ -\frac{i\gamma}{4\pi^4}(\bar{g}_2 + 2\bar{g}_1)\bar{g}_2 & \epsilon - \frac{i\gamma}{4\pi^4}(3\bar{g}_2^2 + \bar{g}_1^2 + \bar{g}_3^2 + 2\bar{g}_1\bar{g}_2 + 2\bar{g}_2\bar{g}_3) & -\frac{i\gamma}{4\pi^4}\bar{g}_2(\bar{g}_2 + 2\bar{g}_3) \\ 0 & -\frac{i\gamma}{4\pi^4}(3\bar{g}_2^2 + 2\bar{g}_2\bar{g}_3) & \epsilon - \frac{i\gamma}{4\pi^4}\bar{g}_2^2 \end{pmatrix}.$$

Applying the condition $\bar{g}_1 = \bar{g}_3$, we get

$$\beta^k_\sigma = \begin{pmatrix} \epsilon - \kappa\bar{g}_2^2 & -\kappa(2\bar{g}_2\bar{g}_1 + 3\bar{g}_2^2) & 0 \\ \kappa\bar{g}_2(2\bar{g}_1 + \bar{g}_2) & \epsilon - \kappa(3\bar{g}_2^2 + 2\bar{g}_1^2 + 4\bar{g}_1\bar{g}_2) & -\kappa\bar{g}_2(3\bar{g}_3 + \bar{g}_2) \\ 0 & -\kappa(3\bar{g}_2^2 + 2\bar{g}_1\bar{g}_2) & \epsilon - \kappa\bar{g}_2^2 \end{pmatrix}$$

with

$$\kappa = i\gamma/4\pi^4. \tag{27}$$

The eigenvalues of (27) are given by

$$\lambda_1 = \epsilon - \kappa\bar{g}_2^2 \tag{28}$$

and the two roots λ_2, λ_3 of the equation

$$\lambda^2 - \lambda[2\epsilon - (4\bar{g}_2^2 + 2\bar{g}_1^2 + 4\bar{g}_1\bar{g}_2)] + \alpha(g_1, g_2, g_3) = 0 \tag{29}$$

with

$$\alpha = (\epsilon - \kappa\bar{g}_2^2)[\epsilon - \kappa(3\bar{g}_2^2 + 2\bar{g}_1^2 + 4\bar{g}_1\bar{g}_2)] - 2\kappa^2\bar{g}_2(2\bar{g}_1\bar{g}_2 + 3\bar{g}_2^2)(\bar{g}_1 + \bar{g}_2 + \bar{g}_3). \tag{30}$$

The reality condition on the roots of (29) reads

$$(2\bar{g}_2^2 + 2\bar{g}_1^2 + 4\bar{g}_1\bar{g}_2)^2 + 8\kappa^2\bar{g}_2(2\bar{g}_1\bar{g}_2 + 3\bar{g}_2^2)(\bar{g}_1 + \bar{g}_2 + \bar{g}_3) > 0$$

or

$$4(\bar{g}_1 + \bar{g}_2)^2 + 8\kappa^2\bar{g}_2(2\bar{g}_1\bar{g}_2 + 3\bar{g}_2^2)(\bar{g}_1 + \bar{g}_2 + \bar{g}_3) > 0.$$

Evaluating this at the non-trivial fixed point $\bar{g}_1 = \bar{g}_3 = x\bar{g}_2$, we see that the above condition is really satisfied. So there is a non-trivial stable fixed point and the field theory is not asymptotically free. But the curious feature about the whole thing is that the solution of the matrix is $O(\sqrt{\epsilon})$; the reason why is not understood very well. Furthermore, at both the trivial and non-trivial fixed points the anomalous dimensions can be computed from equation (18) and are seen to reproduce those of the free Thirring model as $g_2 \to 0$.

References

Ansel'm A A 1959 *Sov. Phys.-JETP* **36** 608

Bogoliubov N N and Parasiuk 1957 *Acta Math.* **97** 227

Bogoliubov N N and Shirkov D V 1959 *Introduction to the Theory of Quantized Fields* (New York: Interscience)

Callen Jr C G 1970 *Phys. Rev.* D **2** 154

Dashen R and Frishman Y 1973 *Phys. Lett.* **46B** 439

Dyatlov I T, Sudakov V V and Ter-Martirosyan K A 1957 *Sov. Phys.-JETP* **5** 631

Hagen C R 1967 *Nuovo Cim.* B **51** 169

't Hooft G and Veltman M 1972 *Nucl. Phys.* B **44** 189

Meyer S and Geieke J 1973 *Phys. Lett.* **44B** 85

Symanzik K 1971 *Commun. Math. Phys.* **23** 49

Zimmermann W 1970 *Lectures on Elementary Particles and Quantum Field Theory, Brandeis Summer School* vol 1, eds S Deser, M Grisaru and H Pendleton (Cambridge, Mass.: MIT Press) chap. 4